## Probability distribution (1)

In probability theory and statistics, a probability distribution is the mathematical function that gives the probabilities of occurrence of different possible outcomes for an experiment. It is a mathematical description of a random phenomenon in terms of its sample space and the probabilities of events.

### Random variable

A random variable is a type of variable in statistics whose possible values depends on the outcome of a certain random phenomenon. It is commonly denoted by X.

Each variables possess a specific probability distribution function

There are two types of Random variable

1. Discrete [discontinuous] variable
2. Continuous variable

(1) Discrete variables → A discrete random variable is a variable whose values take only a finite number of values e.g → the number of students in a college, the number of defective mangoes in a basket of mangoes. The number of accident taking place on a busy road.

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(2) Continuous variables → A random variables is called the continuous if its possible values contain a whole interval of Numbers. e.g → wt of person, Height and length of clothes.

**Discrete variables**

① Probability mass function

$f(x) \geq 0$

$\Sigma f(x) = 1$

② Binomial distribution

③ Poisson distribution

**Continuous variables**

① Probability density function

(a) $f(x) \geq 0$

(b) $\int_{-\infty}^{\infty} f(x) = 1$

② Exponential

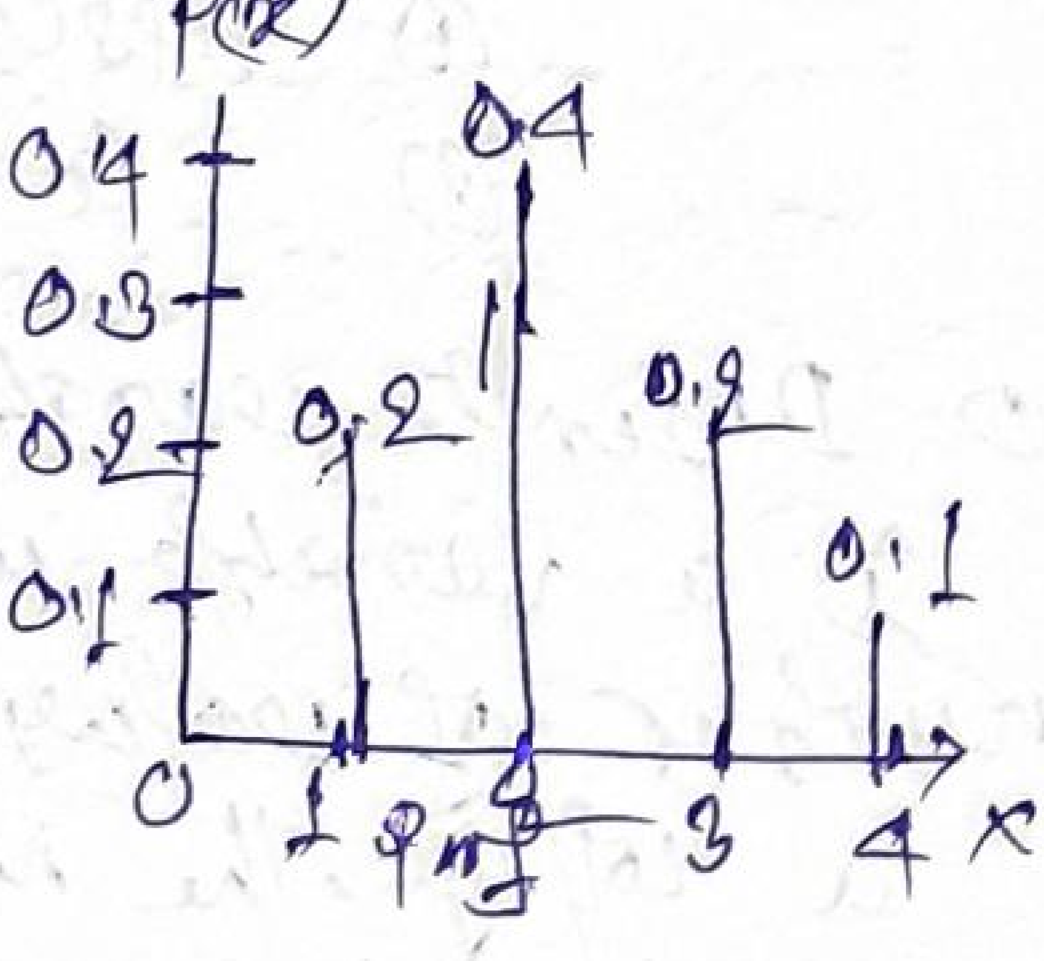
③ Normal distribution

① Probability mass function (pmf) - pmf is a function that gives the probability that a discrete random variable is exactly equal to some value.

CDF (cumulative Distribution function)
$= \Sigma pdf$

Ex: ①

| Random variable (X) | P[x=2] | P[x≤2] |
|---|---|---|
| 0 | 0.10 | |
| 1 | 0.20 | |
| 2 | 0.40 | |
| 3 | 0.20 | |
| 4 | 0.10 | |





then find the cumulative mass function
$P[x<1]$ $P[x<2]$, $P[x>2]$

$P[x<1] = P[0] + P[1]$
$= 0.1 + 0.2$
$= 0.3$

$P(x<2) = P(0) + P(1) + P(2)$
$= 0.1 + 0.2 + 0.4$
$= 0.7$

$P(x>2) = 1 - P[x<2]$
$= 0.3$.

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Q → Let $X$ be a random variable, and $P[X=x]$ is the pmf given by.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(X=x)$ | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2+k$ |

(i) Determine the values of $k$

(ii) Find the probability (i) $P[X\leq 6]$ (ii) $P(3<x\leq 6)$

Solution :→ we know that

$\sum P(x_i) = 1$

Therefore, $P(x=0)+P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)$
$+P(x=6)+P(x=7) = 1$

$\Rightarrow 0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$

$\Rightarrow 9k+10k^2 = 1$

$\Rightarrow 10k^2+9k-1 = 0$

$\Rightarrow 10k^2+10k-k-1 = 0$

$\Rightarrow 10k[k+1]-1[k+1] = 0$

$\Rightarrow [10k-1][k+1] = 0$

$\Rightarrow k = \frac{1}{10}, -1$, Then the values of $k = \frac{1}{10}$

(i) $P[x\leq 6] = 1-P[x>6]$

$= 1-P[7]$

$= 1-(7k^2+k)$

$= 1-\frac{7}{100}-\frac{1}{10}$

$= \frac{100-7-10}{100} = \frac{83}{100}$

(b) $P[3 \le x \le 6] = P(x=4) + P(x=5) + P(x=6)$

$= 3k + k^2 + 2k^2 \quad \left[k = \frac{1}{10}\right]$

$= \frac{3}{10} + \frac{1}{100} + \frac{2}{100}$

$= \frac{30+1+2}{100}$

$= \frac{33}{100}$

Expected values of discrete random variable

If $x$ is a discrete random variables which takes mutually exclusive values $x_1, x_2, \ldots x_n$ with associated probabilities function $P(x_1), P(x_2) \ldots P(x_n)$

Then Expected values $E(x) = \Sigma x_i P(x_i)$

variance of discrete random variables $= E[x^2] - [E(x)]^2$

Q → The cost of food product is Rs 1 per piece while it can be sold for Rs 3 per piece. It is however perishable and any goods unsold at the end of the day are a dead loss. The demand of the product with respective probability is given below

| No of pieces demanded | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|
| probability | 0.07 | 0.10 | 0.23 | 0.38 | 0.12 | 0.10 |

Calculate the expected net profit or loss of 12 pieces are manufactured

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Solution

| unit demanded | Profit |
|---|---|
| 10 | $10\times3-12\times1=18$ |
| 11 | $11\times3-12\times1=21$ |
| 12 | $12\times3-12\times1=24$ |
| 13 | $12\times3-12\times1=24$ |
| 14 | $12\times3-12\times1=24$ |
| 15 | $12\times3-12\times1=24$ |

Expected of profit $= 18\times0.07+21\times0.1+24\times0.23+24\times0.38+24\times0.12+24\times0.1$

$= 1.26+2.1+5.52+9.12+2.88+2.4$

$= 23.28$ Rs

Q→ If x denotes the number of points on a dice. Find the expectation of X. Also find the variance of the probability of distribution i.e find E[{X+E] $E(x^2)-[E(x)]^2 = var(x)$

~~Solution~~

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$: | 1 | 2 | 3 | 4 | 5 | 6 |
| Probability: | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

Soln $E[x] = 1\times\frac{1}{6}+2\times\frac{1}{6}+3\times\frac{1}{6}+4\times\frac{1}{6}+5\times\frac{1}{6}+6\times\frac{1}{6}$

$= \frac{1}{6}+\frac{1}{3}+\frac{1}{2}+\frac{2}{3}+\frac{5}{6}+1$

$= \frac{3+6+9+12+15+18}{18}$

$= \frac{\cancel{63}\,7}{\cancel{18}\,2} = 3.5$

$E[x^2] = 1^2\times\frac{1}{6}+2^2\times\frac{1}{6}+3^2\times\frac{1}{6}+4^2\times\frac{1}{6}+5^2\times\frac{1}{6}+6^2\times\frac{1}{6}$

$= \frac{1}{6}+\frac{4}{6}+\frac{9}{6}+\frac{16}{6}+\frac{25}{6}+\frac{36}{6}$

$= \frac{91}{6}$ $\quad V(x) = \frac{91}{6}-(3.5)^2$

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